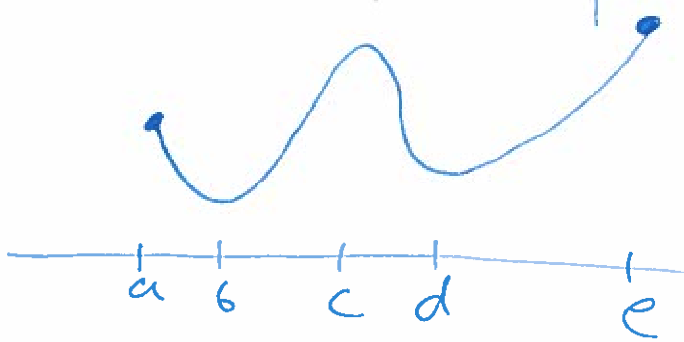
Chapter 4: Applications of Derivatives

4.1: Extreme Values of Functions

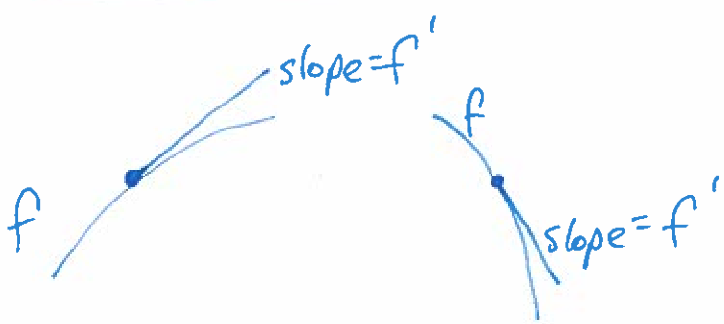
Def^n: Global	Local
Global/Absolute Maximum/Minimum	Local Maxima/Minima
Extreme Values	Local Extrema



a, c, e are local maxima
b, d are local minima
b is global min
e is global max

<u>Extreme Value Theorem:</u> If f is continuous on a closed interval then it attains its global maximum and global minimum.

Relation to Derivative

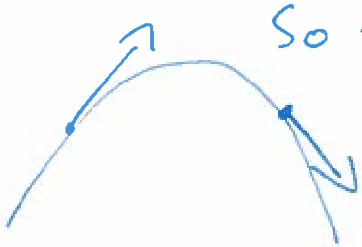


f is increasing $\iff f' > 0$

f is decreasing $\iff f' < 0$

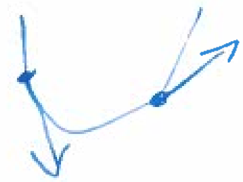
Max

f inc. then dec.
So $f'>0$ then <0



Min

f is dec $\Rightarrow$ inc.
So $f'<0$ then $f'>0$.



By IVT, $f'=0$ at all relative extrema.

<u>Theorem</u>: If x is a relative extrema of f then $f'(x)=0$ or is undefined.

<u>Def^n</u>: If $f'(x)=0$ or is undefined then x is a <u>critical point</u> of f.

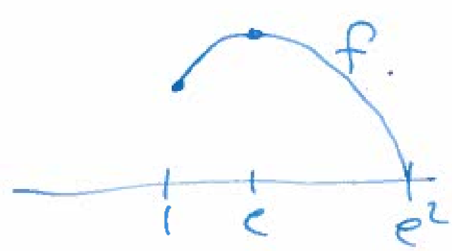
<u>Ex 1</u>: Find global max and min of $f(x)=10x(2-\ln x)$ on $[1,e^2]$.

$f'(x)=10(x\cdot(-\frac{1}{x})+(2-\ln x))=10(1-\ln x)$.

$f'(x)=0$ when $x=e$. So critical pts at $x=1,e,e^2$ (endpoints: 1, e^2)

~~f(e)~~ $f(1)=20$, $f(e)=10\cdot e$, $f(e^2)=0$

So e is global max and e^2 is global min



4.2: Mean Value Theorem

Rolle's Theorem: If f is cont. on $[a,b]$ and differentiable on (a,b) and $f(a)=f(b)$ then there exists some $c \in (a,b)$ such that $f'(c)=0$.

Ex(1): Show $x^3+3x+1=0$ has exactly one solution.

$f(x)=x^3+3x+1$. $f(-1)=-3$ and $f(0)=1$. Thus, by IVT, there exists a solution in the interval $(-1,0)$.

If there was another solution, then we have two points c_1, c_2 such that $f(c_1)=0=f(c_2)$ and by Rolle's Thm there is some point c such that $f'(c)=0$.

However, $f'(x)=3x^2+3=3(x^2+1)$ is never zero.

So we cannot have a second solution.

Mean Value Theorem: Suppose $f(x)$ is cont. on $[a,b]$ and differentiable on (a,b). Then there is some $c \in (a,b)$ such that

$$f'(c)=\frac{f(b)-f(a)}{b-a}.$$

Ex(2): $f(x)=x^2$ on $0\le x\le 2$.

By MVT, there is some $c\in(0,2)$ such that

$$f'(c)=\frac{f(2)-f(0)}{2-0}=2.$$

But clearly $f'(x)=2x$ so that $c=1$. However, it is not always easy to solve for c algebraically.

Physically: MVT says that there is some point in time where the instantaneous rate of change is equal to the average rate of change.

Corollary 1: If $f'(x)=0$ for all $x\in[a,b]$ then $f(x)=C=$ constant on $[a,b]$.

Corollary 2: If $f'(x)=g'(x)$ for all $x\in(a,b)$ then $f(x)=g(x)+C$ on $[a,b]$.

Ex(3): Find the graph $f(x)$ whose derivative is $\sin x$ and goes through the point $(0,2)$.

Answer: $-\cos x+3$

Remark: We can now use these corollaries to prove all the log and exp. Laws.